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ANALYSIS OF THE EFFECT OF PHASE MARGIN ON OUTPUT IMPEDANCE MINIMIZATION OF AN AUTOMOTIVE BUCK CONVERTER

Yabefa, Ebimobowei T., and Osarumwense, Egwaile J.
Department of Electrical/Electronic Engineering,
University of Benin, Benin City, Nigeria
Email: eyabefa@gmail.com

ABSTRACT

Although linear regulators are still seldomly used as post regulators, power supply designers have shifted to the switch mode power supplies (SMPS) due to their higher efficiency and minimal size. In the design of the SMPS, minimization of the output impedance is an important requirement, and is the function of the compensator [1]. The phase margin is a key parameter for the compensator design, and is suggested to be between 45 and 60 degrees [1-2]. The result shows that the output impedance and the phase margin are inversely related. And therefore, in designing the compensator for the output impedance minimization, the phase margin is deliberately selected above 60 degrees.

List of Notations

M_p = Percentage overshoot
φ = Phase margin
Q = Quality factor at crossover
ω_c = Crossover frequency
$Z_{OUT,OL}$ = Open loop output impedance
$Z_{OUT,CL}$ = Closed loop output impedance
$T(s)$ = loop gain of the system

INTRODUCTION

In the design of buck converters, the minimization of the closed loop output impedance is one of the objectives of the loop gain. While the closed loop impedance is both a function of the open loop output impedance, and the magnitude of the closed loop gain, the phase margin at crossover is the main determining design parameter to effectively minimize the output impedance for the entire bandwidth of the converter. References [1],[2],[3],[6] suggests that for the design of the controller, the phase margin is between 45 and 60 degrees. But this range of phase margin does not contribute to minimizing the output impedance of the converter.

For the efficient design of the compensator selected, the resonance in the open loop output impedance is shifted from the resonant frequency to the crossover frequency. Although at crossover, some resonance or overshoot are unavoidable, the overshoot must be within the limits of ensuring that the closed loop output impedance is minimal and not greater than that of the open loop output impedance.

Although, [4] and [6] had mentioned the relationship between the phase margin and the quality coefficient, the effect of the closed loop phase margin and the quality factor on the output impedance minimization have not been discussed. In this paper, the effect of the phase margin on both the quality factor, the percentage overshoot, and the output impedance is explored. The results could facilitate power supply designer's decision in the development of switching power converters with very tight specifications on output impedance and voltage undershoot.

METHODS AND MATERIALS

Section I: Open Loop Output Impedance

The load current -to – output transfer function, also known as the open loop output impedance (z_p) of the converter is obtained from the small signal model, when $sL, \frac{1}{sC}, and\ R$ are all connected in parallel [1],[7].

$$z_p = \frac{sL\ \left(\frac{R}{1+sRC}\right)}{sL+\frac{R}{(1+sRC)}} \quad (1)$$

Therefore,

$$z_p = sL\left[\frac{1}{s^2\ LC+\frac{L}{R}s+1}\right] \quad (2)$$

Comparing equation (2) with the standard second order system of equations (3),the resonant frequency and the quality coefficient are obtained.

$$\frac{1}{\frac{s^2}{{\omega_o}^2}+\frac{s}{\omega_o Q}+1} \quad (3)$$

$$\omega_o = \frac{1}{\sqrt{LC}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

$$Q = R\sqrt{C/_L} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

where w = operating frequency in radians
L = inductance of the inductor
C = capacitance of the capacitor
R = load
Q = the quality factor

As a buck converter is usually modelled as a second order system [3],[4] equation (6) [5] relating the quality factor to the system overshoot, and as shown in Table 1 is very important.

$$M_p(\%) = e^{-\frac{\delta\pi}{\sqrt{1-\delta^2}}} X\,100 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

Where

$$\delta = \frac{1}{2\,Q}$$

Table 1: quality factor and overshoot relation

Overshoot (%)	Quality factor (Q)
23.45	1.1892
21.19	1.1292
19.12	1.0732
17.12	1.0208
15.17	0.9714
13.27	0.9246
11.42	0.8799
9.64	0.8372
8.77	0.8165
7.92	0.7961
6.27	0.7562
4.71	0.7173

Section II: Crossover Phase Margin

Reducing the quality factor of the system, is to select a suitable phase margin at crossover, so that both the Q and the corresponding overshoot are within specification. Using the two poles system of equation (7) [4], [6] at crossover, the closed loop gain of the converter is as shown in equation (8)

$$T(s) = \frac{1}{\left(\frac{s}{\omega_o}\right)\left(1+\frac{s}{\omega_2}\right)} \text{ .... (7)}$$

$$\frac{T(s)}{1+T(s)} = \frac{\frac{1}{\left(\frac{s}{\omega_o}\right)\left(1+\frac{s}{\omega_2}\right)}}{1+\frac{1}{\left(\frac{s}{\omega_o}\right)\left(1+\frac{s}{\omega_2}\right)}} = \frac{1}{\frac{s^2}{\omega_o\omega_2}+\frac{s}{\omega_o}+1} \text{ .... (8)}$$

Comparing equation (8) with equation (9) [4], the quality factor (Q) at the vicinity of the crossover is obtained as shown in (10).

$$\frac{1}{\frac{s^2}{\omega_r^2}+\frac{s}{\omega_r Q}+1} \qquad (9)$$

$$Q = \sqrt{\frac{\omega_o}{\omega_2}} \text{ .... } \qquad (10)$$

From equation (7), at the crossover, where the gain is 0dB,

$$\left|\frac{1}{\left(\frac{j\omega_c}{\omega_o}\right)\left(1+\frac{j\omega_c}{\omega_2}\right)}\right| = 1 \text{ .... (11)}$$

So that,

$$\left|\left(\frac{j\omega_c}{\omega_o}\right)\left(1+\frac{j\omega_c}{\omega_2}\right)\right| = \left|\left(\frac{j\omega_c}{\omega_o}\right)\right|\left|\left(1+\frac{j\omega_c}{\omega_2}\right)\right| = 1 \text{ .... (12)}$$

Therefore,

$$\frac{\omega_c}{\omega_o}\sqrt{1+\left(\frac{\omega_c}{\omega_2}\right)^2} = 1 \text{.................................... (13)}$$

From (13)

$$\left(\frac{\omega_c}{\omega_o}\right)^2\left(1+\left(\frac{\omega_c}{\omega_2}\right)^2\right) = 1 \quad \text{ .... (14)}$$

but $\omega_o = Q^2\omega_2$ from equation (10).

Therefore,

$$\left(\frac{\omega_C}{Q^2\omega_2}\right)^2\left(1+\left(\frac{\omega_C}{\omega_2}\right)^2\right)=1$$

$$\frac{\omega_C{}^2}{Q^4\omega_2{}^2}\left(1+\frac{\omega_C{}^2}{\omega_2{}^2}\right)=1$$

Which is the same thing as equation (15)

$$\frac{\omega_C{}^2(\omega_2{}^2+\omega_C{}^2)}{\left(Q_{\omega_2}\right)^4} = 1 \qquad \text{ .... (15)}$$

Solving for the crossover frequency,

$$\omega_C{}^2(\omega_2{}^2+\omega_C{}^2) = \left(Q_{\omega_2}\right)^4$$

$$\omega_C{}^2\omega_2{}^2+\omega_C{}^4 = \left(Q_{\omega_2}\right)^4$$

$$\omega_C{}^4+\omega_C{}^2\omega_2{}^2-\left(Q_{\omega_2}\right)^4 = 0 \text{ .... (16)}$$

Representing × $as\ \omega_c{}^2$ in (16) and comparing with the quadratic expression of (17), equation (18) is obtained.

$\alpha x^2 + bx + C = 0$ ………………………………….. (17)

$X^2 + \omega_2{}^2 X - \left(Q_{\omega_2}\right)^4 = 0$ ……………………… (18)

Using the solution to equation (17), as shown in (19), and simplifying, equation (20) is got.

$X = \frac{-b \pm \sqrt{b^2 - 4\alpha c}}{2\alpha}$ ……………………….. (19)

$X = \frac{-\omega_2{}^2 \pm \sqrt{(\omega_2{}^2)^2 - 4(1)(-Q\omega_2)^4}}{2}$

Solving the equation in (19) gives equation (20)

$X = \frac{-\omega_2{}^2 \pm \omega_2{}^2\sqrt{1+4Q^4}}{2}$ …………… ……………………………. (20)

But X=$\omega_C{}^2$

therefore,

$\omega_c{}^2 = \frac{-\omega_2{}^2 \pm \omega_2{}^2\sqrt{1+4Q^4}}{2}$ ……………………… (21)

Ensuring that the Q of the system is minimal,

$\omega_c{}^2 = \frac{\omega_2{}^2\sqrt{1+4Q^4}}{2} - \omega_2{}^2$ ………………………………. (22)

So that,

$\omega_c = \frac{\omega_2\sqrt{\left(\sqrt{1+4Q^4} - 1\right)}}{\sqrt{2}}$ ………………………………………….. (23)

In order to determine the phase margin at crossover, reference [4] suggests that;

$\arg T(\omega_c) = -\frac{\pi}{2} - \tan^{-1}\frac{\omega_c}{\omega_2}$ ……………………… (24)

but at crossover,

$arg\ T\ (\omega_c) = -180 + \varphi$ ……………… (25)

From (25),

$\varphi = \pi + \arg(T\omega_c)$ …………………………. (26)

Substituting equation (24) into equation (26),

$\varphi = \pi - \tan^{-1}\frac{\omega_c}{\omega_2} - \frac{\pi}{2} = \frac{\pi}{2} - \tan^{-1}\frac{\omega_c}{\omega_2}$ ……………. (27)

But, from trigonometry, $\tan^{-1}x + \tan^{-1}\frac{1}{x} = \frac{\pi}{2}$........ (28)

Therefore,

$\varphi = \tan^{-1}\frac{\omega_c}{\omega_2} + \tan^{-1}\frac{\omega_2}{\omega_c} - \tan^{-1}\frac{\omega_c}{\omega_2}$ ………………… (29)

So that

$\varphi = \tan^{-1}\frac{\omega_2}{\omega_c}$ ……………………. (30)

from equation (23), $\omega_c = \frac{\omega_2\sqrt{\left(\sqrt{1+4Q^4}-1\right)}}{\sqrt{2}}$

Therefore, $\varphi = \tan^{-1}\left[\sqrt{\frac{2}{\sqrt{(1+4Q^4)}-1}}\right]$………………………… (31)

From (31),

$\sqrt{(1+4Q^4)}\tan^2(\varphi) - \tan^2(\varphi) = 2$

So that,

$$4Q^4 = \frac{4 + 4\,tan^2(\varphi)}{tan^4(\varphi)}$$

Therefore,

$Q = \frac{\sqrt[4]{(1+tan^2(\varphi))}}{\tan(\varphi)}$ …………………………….. (32)

but,

$1 + tan^2(\varphi) = sec^2(\varphi)$ and $sec^2(\varphi) = \frac{1}{cos^2(\varphi)}$ …………… (33)

Therefore,

$Q = \frac{1}{\sqrt{\cos(\varphi)}}\frac{\cos(\varphi)}{\sin(\varphi)}$ ………………… (34)

After simplifying(34), equation (35) is obtained

$Q = \frac{\sqrt{\cos(\varphi)}}{\sin(\varphi)}$ …………………………………………… (35)

Table 2 shows the choice of phase margin and its corresponding Q of the system.

Table 2: phase margin and quality factor at crossover

Phase margin	Quality factor
45	1.1892
47	1.1292
49	1.0732
51	1.0208
53	0.9714
55	0.9246
57	0.8799
59	0.8372
60	0.8165
61	0.7961
63	0.7562
65	0.7173

The quality factor reduces with increasing values of phase margin. The reducing Q values corresponds to reducing percentage overshoot in the system as expressed in equation (1), and also shown in table 1.

Section III: Closed Loop Output Impedance

The closed loop output impedance is related to the open loop output impedance by the closed loop gain of the system [4];

$$Z_{out,CL} = \frac{Z_{out,OL}}{\sqrt{2-2\cos(\varphi)}} \quad \ldots\ldots\ldots \quad (27)$$

The choice of phase margin determines the magnitude of the output impedance as shown in (27), and also dictates the overshoot. Table 3 shows a range of phase margins, and the corresponding impact on typical system performance indicators.

Table 3: phase margin and output impedance relation

Phase Margin	Quality factor	Overshoot (%)	Magnitude of Closed loop gain	$Z_{out,CL}$ and $Z_{out,OL}$ compared
45	1.1892	23.45	1.3066	$Z_{out,CL} > Z_{out,OL}$
47	1.1292	21.19	1.2539	$Z_{out,CL} > Z_{out,OL}$
49	1.0732	19.12	1.2057	$Z_{out,CL} > Z_{out,OL}$
51	1.0208	17.12	1.1614	$Z_{out,CL} > Z_{out,OL}$
53	0.9714	15.17	1.1206	$Z_{out,CL} > Z_{out,OL}$
55	0.9246	13.27	1.0828	$Z_{out,CL} > Z_{out,OL}$
57	0.8799	11.42	1.0479	$Z_{out,CL} > Z_{out,OL}$
59	0.8372	9.64	1.0154	$Z_{out,CL} > Z_{out,OL}$
60	0.8165	8.77	1	$Z_{out,CL} = Z_{out,OL}$
61	0.7961	7.92	0.9851	$Z_{out,CL} < Z_{out,OL}$
63	0.7562	6.27	0.9569	$Z_{out,CL} < Z_{out,OL}$
65	0.7173	4.71	0.9306	$Z_{out,CL} < Z_{out,OL}$

CONCLUSION

In designing switch mode power supplies for the HEVs and the EVs, efficiency and voltage undershoot considerations are paramount. The sustained connection between the SMPS and the load requires minimal voltage undershoot. The specification of the output voltage variation used in determining the capacitor size must be maintained even during step loading. Minimizing the output impedance increases the efficiency of the converter, because, the loss associated with the capacitive reactance is reduced. The choice of the closed loop phase margin determines both the magnitude of the output impedance, and the peaking/ringing of the system. Therefore, minimizing the output impedance is to deliberately select the phase margin above 60 degrees.

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